Carolina Poker Club: Problem Set 1

August Warshauer — aawarshauer@gmail.com
Benjamin Goroshnik — ben.goroshnik@gmail.com

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1 Combinatorics & Probability

1. How many distinct 2 hole card combinations can you make with a standard deck of cards?

2. Fernando is on vacation, sailing the seas in a 30 million dollar yacht. His business is cactus farming. While on vacation, he has nominated his pet cat, Gavin, to take care of raising the next crop of cacti. Gavin is in charge of planting 60 seeds of three different varieties of cacti. But when Fernando returns, he finds Gavin forgot to count how many cacti of each variety have been planted. Knowing Gavin did manage to plant 60 seeds, how many unique combinations of cacti crop could Fernando be currently germinating?

3. Uh oh! A new virus has appeared, called the Fly Flu. There is a 3% chance any individual has contracted this pathogen. Luckily, students at Chapel Hill were quick to react and have developed a test that gives a false positive at a rate of 2% and a false negative at a rate of 7%. Given you tested positive for Fly Flu, what is the likelihood you do **not**, in fact, have the virus?

4. Suppose you had one standard deck of cards. What is more likely: drawing 5 black cards in a row without replacement, or drawing at least 9 black cards in 11 draws with replacement.

5. Bravo! You have just been hired as a developer of a 20 story high-rise building right next to the appropriately named Billionaires Row in New York City. For every floor, you have a decision: you may either develop the entire floor as one "Class A" unit, or you may split the floor into four quarters. These quarters are distinguished as Northwest, Northeast, Southwest, and Southeast. There are two types of quarter units, called "Class B" and "Class C" units. For purposes of this project, floors numbers are distinguishable, specific quarters are distinguishable, but Classes of units are identical. The project supervisor wants to know: how many unique combinations of units is it possible to develop?

6. You like to travel. But you are particular: you only visit Chapel Hill, Dallas, and San Francisco. Every day, you have a 20% chance of choosing to remain in the city you currently reside. If you are in Chapel Hill and you are twice as likely to travel to Dallas than SF. If you are in SF, you are three times as likely to travel to Dallas than Chapel Hill. If you are in Dallas, you are equally likely to travel to Chapel Hill as SF. You begin in Chapel Hill. After 5 days, what is the probability you are still in Chapel Hill?

7. Construct an infinite two dimensional euclidean coordinate plane. In this plane, draw concentric circles centered on the origin with radii $r = 2n$ where $n \in \mathbb{Z}^+$. Next, you are to drop an infinitely thin needle of length 1 at random in this coordinate plane. What is the probability the needle is not touching or crossing any of the concentric circles?

8. BONUS: As a continuation of the previous problem, what is the probability of the needle avoiding any concentric circles given the midpoint of the needle must land further than 10 units from the origin?

2 Poker Theory

1. What are your pot odds facing an effective shove of $90 into a pot of $55?

2. There is initially $200 in the pot. Your opponent then bets $100 on the river. You believe your opponent has you beat at 70% of the time. Should you call?

3. You have just seen the flop and hold air. You bet $10 into a pot of $70. At least how frequently must your opponent call your bet in order to prevent you from always profitably making this bet (Minimum Defence Frequency)?

4. Describe the difference between a semi-bluff and a bluff. Give an example of a scenario in a where a semi-bluff is advantageous. Explain why this strategy can be more effective than a pure bluff or a value bet in this situation.

5. If you are a regular player in $1/$2 no-Limit cash games, what should your bankroll be to comfortably withstand the game's variance? Explain the reasoning behind your answer.

3 Solutions

3.1 Combinatorics & Probability

1. $\binom{52}{2} = 1326$

2. $\binom{62}{2} = 1891$

3. 41%

4. Drawing 9 of 11 (3.27% vs. 2.53%)

5. $\sum_{i=0}^{20} \binom{20}{i} \sum_{j}^{4i} \binom{4i}{j} = 4.064 * 10^{24}$

6. 0.28192 from Markov Chain $\begin{pmatrix} 0.2 & \frac{0.8*2}{3} & \frac{0.8}{3} \\ 0.4 & 0.2 & 0.4 \\ 0.2 & 0.6 & 0.2 \end{pmatrix}^5 = \begin{pmatrix} 0.28192 & 0.41709 & 0.30099 \\ 0.28508 & 0.41134 & 0.30357 \\ 0.28120 & 0.41838 & 0.30040 \end{pmatrix}$

7. This is Buffons Needle Problem (as $r \to \infty$, circles become straight lines) with $l = 1$ & $d = 2$. See wolfram. The solution becomes $1 - \int_0^{\frac{1}{2}} \frac{2 \arccos(2x)}{\pi} \, dx \approx 0.6817$

8. Since plane is not infinite, circles change a lot. Apply law of cosines.
   The solution becomes:
   $P = 1 - \frac{\sum_{j=1}^{5} \frac{2}{\pi} * \int_0^{\frac{1}{2}} [\pi - \arccos(\frac{\frac{1}{2}^2 + (2j-x)^2 - (2j)^2}{2j-x})] \, dx + \sum_{j=1}^{4} \frac{2}{\pi} * \int_0^{\frac{1}{2}} [\arccos(\frac{\frac{1}{2}^2 + (2j+x)^2 - (2j)^2}{2j+x})] \, dx}{10}$
   ≈ 0.71217

3.2 Poker Theory

1. 38.3%

2. Yes (25% pot odds)

3. 87.5%

4. Answers vary

5. 20-30x buy-in is recommended, so standard 100bb 1$/$2 implies $4000+